

# Bounds and implications of neutrino magnetic moments from atmospheric neutrino data

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The neutral current effects of the future high statistics atmospheric neutrino data can be used to distinguish the mechanisms between a  $\nu_\mu$  oscillation to a tau neutrino or to a sterile neutrino. However, if neutrinos possess large diagonal and/or transition magnetic moments, the neutrino magnetic moments can contribute to the neutral current effects which can be studied by the single  $\pi^0$  production events in the Super-K data. This effect should be included in the future analyses of atmospheric data in the determination of  $\nu_\mu$  to tau or sterile neutrino oscillation. [S0556-2821(99)01215-1]

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## I. INTRODUCTION

Neutrinos might possess two properties which are feeble, but will be important barometers of physics beyond the standard model scale. These are neutrino mixings (masses and oscillations) [1] and neutrino magnetic moments [2]. Before 1998, experiments gave bounds on these properties, in general.

But recent results from the Super-Kamiokande Collaboration [3] have provided strong evidence for a deficit in the flux of atmospheric neutrinos, which are presented in the form of the double ratio

$$R = \frac{(N_\mu/N_e)_{obs}}{(N_\mu/N_e)_{MC}}, \quad (1)$$

which implies the existence of  $\nu_\mu$  oscillations. The measured value of  $R$  for Super-Kamiokande is  $0.61 \pm 0.06 \pm 0.05$  for the sub-GeV data and  $0.67 \pm 0.06 \pm 0.08$  for the multi-GeV data, while we expect  $R=1$  in a world without oscillations. The muon neutrino oscillation into another species of neutrino provides a natural explanation for the deficit and even the zenith angle dependence. The  $\nu_\mu \rightarrow \nu_\tau$  oscillation is the most favorable solution for the atmospheric neutrino problem, whereas the  $\nu_\mu \rightarrow \nu_e$  oscillation is strongly disfavored by CHOOZ results [4]. The oscillations into sterile neutrinos ( $\nu_s$ ) give a plausible solution as well [5]. This evidence for neutrino oscillations is also supported by the SOUDAN2 [6] and by the Super-Kamiokande [7] and MACRO [8] data on upward-going muons.

It is usually assumed that the neutral current effect in neutrino oscillation experiments is unchanged, since any

standard model neutrino produced by oscillations has the same neutral current (NC) interaction. Therefore, the ratio of the NC and charged currents (CC) events is important to investigate the neutrino neutral current. Thus the observation of single  $\pi^0$  events, induced by the neutral current, by the Super-Kamiokande Collaboration [3], can lead to important physical implications.

The  $\pi^0$  NC event is detected as two diffuse rings. On the other hand, the CC events due to  $\nu_e$  are detected as one diffuse ring due to  $e^\pm$  and one sharp ring due to  $\pi^\pm$ , and the CC events due to  $\nu_\mu$  are detected as two sharp rings from  $\mu^\pm$  and  $\pi^\pm$  [9]. Thus, a NC event can be discriminated from a  $\nu_e$  CC event and a  $\nu_\mu$  CC event [10,11]. It has been considered difficult to separate NC and CC events clearly. But the single  $\pi^0$  events described above can be used to discriminate NC events from CC events. Indeed, it is believed that the cleanest way to identify NC events in Super-Kamiokande is to detect a single  $\pi^0$  from the process  $\nu + N \rightarrow \nu + N + \pi^0$ , with  $N$  being either a neutron or a proton below the Cherenkov threshold. The  $\pi^0$  is detected via its decay into two photons which lead to two diffuse  $e$ -like rings whose invariant mass is consistent with the  $\pi^0$  mass [9]. The ratio of  $\pi^0$ -like events to  $e$ -like events compared to the same ratio of the Monte Carlo in the absence of the oscillation has been measured by the Super-Kamiokande Collaboration [7]:

$$R_{\pi^0/e} = \frac{(\pi^0/e)_{data}}{(\pi^0/e)_{MC}} = 0.93 \pm 0.07_{stat} \pm 0.19_{sys}, \quad (2)$$

where the systematic error is dominated by the poorly known single  $\pi^0$  cross section, and the statistical error is based on 535 days of running. The ratio  $R_{\pi^0/e}$  is expected to be 1 for  $\nu_\mu$ - $\nu_\tau$  oscillations while 0.75 for  $\nu_\mu$ - $\nu_s$  oscillations or  $\nu_\mu$ - $\nu_e$  oscillations if one takes the measured  $\nu_\mu/\nu_e$  ratio to be 0.65 [10]. The admixture of  $\nu_\mu$ - $\nu_\tau$  and  $\nu_\mu$ - $\nu_e$  oscillations also leads to a deviation of  $R_{\pi^0/e}$  from 1. Therefore, a precise

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measurement of the ratio will be used to distinguish  $\nu_\mu \rightarrow \nu_\tau$  from  $\nu_\mu \rightarrow \nu_s$  oscillation.

At first sight, it is likely that any deviation of  $R_{\pi^0/e}$  from 1 implies muon neutrino oscillations into a sterile neutrino. However, if there exists a large muon neutrino magnetic moment (diagonal or transition), it will produce an additional neutral current effect which has to be separated out to draw a definite conclusion. Indeed, right after the discovery of the neutral current, the upper bound on the muon neutrino magnetic moment was given [12]. Also, the experimental bounds on transition magnetic moments and other properties in view of NC data were presented [13].

The theoretical problem of obtaining a large neutrino magnetic moment has begun with interactions beyond the standard model [2]. In general, the loop diagram will have a (mass)<sup>2</sup> suppression, presumably by  $M_X^2$ , where  $M_X$  can be the  $W$  boson mass or a scalar mass. It is possible to have a large Dirac neutrino magnetic moment if the loop contains a heavy fermion [2],

$$\mu_\nu \sim \frac{m}{M_X^2}, \quad (3)$$

where  $m$  is the mass of the heavy fermion. This mechanism can be generalized in models with scalars [14].

But the same loop without an external photon line would give a contribution to the neutrino mass matrix. Therefore, one expects, taking the coupling as  $10^{-3}$ ,

$$\mu_\nu \sim 10^{-3} \frac{m_e}{M_X} \left( \frac{m_\mu}{M_X} \right)^{1/3} \mu_B \sim 10^{-13} \mu_B, \quad (4)$$

where  $\mu_B = e\hbar/2m_e c$  is the electron Bohr magneton and we used  $m_\nu \sim O(1)$  eV for the numerical illustration. To suppress the contribution to the mass and still allow a large magnetic moment, continuous [15] and discrete [16] symmetries have been considered. In this case, the neutrino magnetic moment can be as large as  $\mu_{\nu_\mu} \sim 10^{-10} \mu_B$ , which is not affected by the SN 1987A constraint  $\sim 10^{-13} \mu_B$  [17] since this bound applies to the electron neutrino only.

However, a large transition magnetic moment to a sterile neutrino is not forbidden that severely. For example, one can introduce a transition moment with an accompanying mass as large as several hundred MeV. Of course, the masses of the light neutrinos are bounded by a few eV. In this case, the transition neutrino magnetic moments can be as large as  $10^{-7} \mu_B$  and may contribute to NC events. In particular, we are interested in single  $\pi^0$  production through a large transition magnetic moment, which would contribute to  $R_{\pi^0/e}$ . In this spirit, we will obtain the upper bound on the transition neutrino magnetic moment (to a sterile neutrino) from  $R_{\pi^0/e}$ .

On the other hand, the experimental bound of the transition magnetic moment is  $\mu_{\nu_\mu} \lesssim 10^{-9} \mu_B$  [18]. This is obtained from the  $\nu_\mu$  neutral current experiments because the sterile neutrino  $\nu_s$  can be freely produced if the mass difference of  $\nu_\mu$  and  $\nu_s$  is much smaller than the center of mass energy in the process [13,19].

This paper is organized as follows: In Sec. II we describe the amplitude for the single  $\pi^0$  production. In Sec. III, the kinematics and the differential cross section for single  $\pi^0$  production are given. In Sec. IV, we present the contribution to the cross section of the  $\pi^0$  production generated by a possible neutrino transition magnetic moment. In Sec. V, we discuss the physical implications based on the numerical result.

## II. PRODUCTION OF THE SINGLE NEUTRAL PION

Single  $\pi^0$  production has two contributions: one from the production and decay of the (3/2,3/2) baryon resonances and the other from the continuum contribution. At low energies ( $E < 2$  GeV), the contribution from baryon resonance production is a dominant one for single  $\pi^0$  production [20]:

$$\begin{aligned} \nu + N &\rightarrow \nu + N^*, \\ N^* &\rightarrow \pi^0 + N, \end{aligned} \quad (5)$$

where  $N^*$  represents baryon resonances. The cross section for single  $\pi^0$  production in the region  $W < 1.6$  GeV/ $c^2$  ( $W$  is the hadronic invariant mass in the final state) can be described following Fogli and Nardulli [20]. The effective Lagrangian for the neutrino neutral current is defined by

$$\mathcal{L}_{NC} = \frac{1}{\sqrt{2}} G_F \bar{\nu} \gamma^\lambda (1 + \gamma_5) \nu J_\lambda^{NC}, \quad (6)$$

by assuming the following general  $V,A$  structure of the hadronic part of the NC:

$$J_\lambda^{NC} = g_V^3 V_\lambda^3 + g_A^3 A_\lambda^3 + g_V^8 V_\lambda^8 + g_A^8 A_\lambda^8 + g_V^0 V_\lambda^0 + g_A^0 A_\lambda^0, \quad (7)$$

where  $V_\lambda^i, A_\lambda^i$  ( $i=3,8,0$ ) are the SU(3) nonet partners of the CC [21]. Neglecting the strange and charm NC's, one can write

$$J_\lambda^{NC} = g_V V_\lambda^3 + g_A A_\lambda^3 + g_V' V_\lambda'^0 + g_A' A_\lambda'^0, \quad (8)$$

where

$$V_\lambda'^0 = \sqrt{\frac{1}{3}} (V_\lambda^8 + \sqrt{2} V_\lambda^0), \quad (9)$$

$$A_\lambda'^0 = \sqrt{\frac{1}{3}} (A_\lambda^8 + \sqrt{2} A_\lambda^0), \quad (10)$$

the electromagnetic current being given by  $J_\lambda^{em} = V_\lambda^3 + \frac{1}{3} V_\lambda'^0$ . From the Weinberg-Salam model [21],

$$g_V = \frac{1}{2} - \sin^2 \theta_W, \quad g_A = \frac{1}{2}, \quad (11)$$

$$g_V' = -\frac{1}{3} \sin^2 \theta_W, \quad g_A' = 0. \quad (12)$$

The isospin decomposition of one  $\pi^0$  channels is given by

$$A(\nu p \rightarrow \nu p \pi^0) = \frac{1}{3}(2A_3 + A_1) + \sqrt{\frac{1}{3}}S, \quad (13)$$

$$A(\nu n \rightarrow \nu n \pi^0) = \frac{1}{3}(2A_3 + A_1) - \sqrt{\frac{1}{3}}S. \quad (14)$$

The reduced matrix elements  $A_1, A_3$  are given by

$$A_3 = \frac{1}{\sqrt{2}}(A_\Delta^0 + A_\pi^0 + A_N^0), \quad (15)$$

$$A_1 = \frac{3}{2\sqrt{2}}A_{NN\pi}^0 - \sqrt{2}A_\pi^0 - \frac{1}{2\sqrt{2}}A_N^0 + A_S^0 + A_P^0 + A_D^0, \quad (16)$$

where  $A_i^0, i=N, NN\pi, P, S, D$  are given in the Appendix and the indices  $S, P, D$  denote  $S_{11}, P_{11}, D_{11}$  [22], respectively. The contributions to the amplitude  $S$  come from the following amplitudes:

$$S = \frac{1}{2}\sqrt{3} \left( \sqrt{\frac{1}{2}}A_N^0 + \sqrt{\frac{1}{2}}A_{NN\pi}^0 + \frac{2}{3}A_P^0 + \frac{2}{3}A_S^0 + \frac{2}{3}A_D^0 \right). \quad (17)$$

As is well known, the dominant contribution to the amplitude  $A$  comes from the  $\Delta$  resonance in this region ( $W < 1.6 \text{ GeV}/c^2$ ) [23]. Then, the amplitude for the single  $\pi^0$  production can be described by

$$A_{NC} \approx \frac{G_F}{\sqrt{2}} l_\alpha J^\alpha, \quad (18)$$

where

$$l_\alpha = \bar{u}(k') \gamma_\alpha (1 + \gamma_5) u(k), \quad (19)$$

$$J^\alpha = \frac{g}{m_\pi} \bar{u}(p') q_\pi^\rho D_{\mu\rho} \left[ - (g^{\mu\alpha} \not{q} - q^\mu \gamma^\alpha) \gamma_5 g_V \frac{C_3^V}{M_N} - (g^{\mu\alpha} q \cdot p_\Delta - q^\mu p_\Delta^\alpha) \gamma_5 g_V \frac{C_4^V}{M_N^2} - g^{\mu\alpha} g_A C_5^A \right] u(p), \quad (20)$$

where  $M_N$  is the nucleon mass,  $D_{\mu\rho}$  is the propagator of Rarita-Schwinger field which is given by

$$D_{\mu\rho}(p) = \frac{\not{p} + M'}{p^2 - M'^2 + iM'\Gamma} \left( g_{\mu\rho} - \frac{2}{3} \frac{p_\mu p_\rho}{M'^2} + \frac{1}{3} \frac{p_\mu \gamma_\rho - p_\rho \gamma_\mu}{M'} - \frac{1}{3} \gamma_\mu \gamma_\rho \right), \quad (21)$$

and  $q = p_\Delta - p = k - k'$  is the momentum transfer,  $p_\Delta = p' + q_\pi$ ,  $\Gamma$  is the decay width,  $C_i^V$  and  $C_i^A (i=3,4,5)$  are the vector and axial vector transition form factors as defined by

Llewellyn-Smith [24], and  $M'$  is mass of the  $\Delta$  resonance. As shown in Ref. [20], the form factors  $C_i(q^2)$  can be obtained by comparison with the values of the helicity amplitudes given by the relativistic quark model [25]. The explicit forms are given by

$$C_3(q^2) = \frac{1.7 \sqrt{1 - q^2/4M_R^2}}{[1 - q^2/(M_R + M_N)^2]^{3/2} [1 - q^2/0.71 \text{ GeV}^2]^2}, \quad (22)$$

$$C_4(q^2) = - \frac{M_N}{\sqrt{W^2}} C_3(q^2), \quad (23)$$

$$C_5(q^2) = 0. \quad (24)$$

Then, the vector form factors  $C_i^V(q^2)$  used in Eq. (20) are given by

$$C_i^V(q^2) = \sqrt{3} C_i(q^2). \quad (25)$$

The axial form factor  $C_i^A(q^2)$  is also taken to be the general formula

$$C^A(q^2) = \frac{C^A(0)}{(1 - q^2/M_A^2)^2}, \quad (26)$$

where  $M_A = 0.65 \text{ GeV}/c^2$  for  $C_5^A$  and  $M_A = 1.0 \text{ GeV}/c^2$  for the other resonances.

### III. KINEMATICS

Consider the process given in Eq. (5). It is convenient to choose the center of momentum frame. Without loss of generality, we can choose the initial four momenta of the neutrino and the nucleon in the c.m. frame as  $(p, p, 0, 0)$  and  $(E_N, -p, 0, 0)$ , respectively, where

$$p = \frac{E_\nu}{\sqrt{1 + 2E_\nu/M_N}}, \quad (27)$$

$$E_N = \frac{M_N + E_\nu}{\sqrt{1 + 2E_\nu/M_N}}, \quad (28)$$

where  $E_\nu$  is the incident neutrino energy in the laboratory frame. The final four-momenta of neutrino, nucleon, and  $\pi^0$  are  $k'$ ,  $p'$ , and  $q_\pi$ , respectively, where

$$k' = E_{\nu'} [1, (\cos \theta, \sin \theta, 0)], \quad (29)$$

$$p' = E_{N'} \left[ 1, \sqrt{1 - \frac{M_N^2}{E_{N'}^2}} (-\cos \beta \cdot \cos \theta + \sin \beta \cdot \sin \theta \cdot \cos \phi, -\cos \beta \cdot \sin \theta - \sin \beta \cdot \cos \theta \cdot \cos \phi, -\sin \beta \cdot \sin \phi) \right], \quad (30)$$

$$q_\pi = E_\pi \left[ 1, \sqrt{1 - \frac{m_\pi^2}{E_\pi^2}} (-\cos \alpha \cdot \cos \theta - \sin \alpha \cdot \sin \theta \cdot \cos \phi, -\cos \alpha \cdot \sin \theta + \sin \alpha \cdot \cos \theta \cdot \cos \phi, \sin \alpha \cdot \sin \phi) \right], \quad (31)$$

where

$$\cos \alpha = \frac{\vec{k}'^2 - \vec{p}'^2 + \vec{q}_\pi^2}{2|\vec{k}'| \cdot |\vec{q}_\pi|}, \quad (32)$$

$$\cos \beta = \frac{\vec{k}'^2 + \vec{p}'^2 - \vec{q}_\pi^2}{2|\vec{k}'| \cdot |\vec{p}'|}. \quad (33)$$

The angles  $\theta$  and  $\phi$  correspond to rotations around the  $z$  and  $x$  axes, respectively. Then, the differential cross section  $d\sigma$  can be expressed as

$$d\sigma = \frac{(2\pi)^4 |\overline{A_{NC}}|^2}{4(p \cdot k)} d\Phi_3 = \frac{|\overline{A_{NC}}|^2}{32(2\pi)^4 M_N E_\nu} dE_{\nu'} dE_\pi d\phi d(\cos \theta), \quad (34)$$

where

$$|\overline{A_{NC}}|^2 = \frac{G_F^2}{2} L_{\mu\nu} J^{\mu\nu}, \quad (35)$$

with

$$L_{\mu\nu} = \frac{1}{2} \sum_{\text{spins}} l_\mu^\dagger l_\nu, \quad (36)$$

and

$$J_{\mu\nu} = \frac{1}{2} \sum_{\text{spins}} J_\mu^\dagger J_\nu, \quad (37)$$

where the summation is performed over the hadronic spins.

#### IV. CONTRIBUTION FROM THE NEUTRINO MAGNETIC MOMENT

In this section, we consider the  $\Delta$  production arising from the Feynman diagram shown in Fig. 1. The decay of  $\Delta$  to a

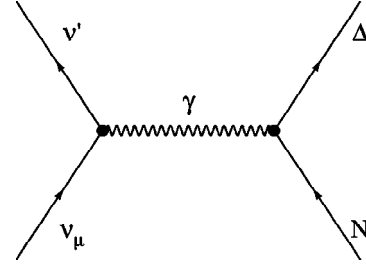


FIG. 1. Feynman diagram for the  $\Delta$  production arising from the neutrino transition magnetic moment.

nucleon plus  $\pi^0$  follows in the detector, and one observes the  $\nu' + N + \pi^0$  final state. The  $\nu_\mu - \nu' - \gamma$  vertex is parametrized by a transition magnetic moment

$$i f_{\nu' \nu_\mu} \mu_B \bar{u}(l')_{\nu'} \sigma_{\mu\nu} q^\nu u(l)_{\nu_\mu}, \quad (38)$$

where  $q = l - l' = p' + q_\pi - p$  is the momentum transfer. The coupling  $f_{\nu' \nu_\mu}$  at  $q^2 = 0$  is the transition neutrino magnetic moment in units of the electron Bohr magneton and will be denoted as  $f'$ .

The squared matrix element that describes the single  $\pi^0$  production, induced by a transition neutrino magnetic moment  $f'$ , can be written as

$$|\overline{A_M}|^2 = \frac{f'^2 \mu_B^2}{q^4} M^{\mu\nu} J_{\mu\nu}^{em}, \quad (39)$$

where

$$M^{\mu\nu} = \frac{1}{2} \sum_{\text{spins}} [\bar{u}(l') \sigma^{\mu\alpha} q_\alpha u(l)] [\bar{u}(l) \sigma^{\nu\beta} q_\beta u(l')] \quad (40)$$

and

$$J_{\mu\nu}^{em} = \frac{1}{2} \sum_{\text{spins}} J_\mu^{em} J_\nu^{em}, \quad (41)$$

where  $J_{\mu\nu}^{em}$  is the hadronic electromagnetic current given before by  $J_\mu^{em} = V_\mu^3 + (1/3)V_\mu^0$ .

#### V. RESULTS AND DISCUSSIONS

In this section we present the numerical results of the cross section of single  $\pi^0$  production for the NC interactions. The calculated cross section generated by the neutrino transition magnetic moment is shown in Fig. 2 as a function of the incident neutrino energy for a hadronic invariant mass less than  $1.6 \text{ GeV}/c^2$ . We find that the values of the cross section are of the order  $10^{-40} \text{ cm}^2$  for  $E_\nu \leq 2 \text{ GeV}$ .

In order to see how the contribution to the cross section generated by the neutrino magnetic moment can be constrained by the experimental results of the ratio  $R_{\pi^0/e}$ , it is sufficient to calculate the ratio  $\sigma_{f'}/\sigma_0^{NC}$  where  $\sigma_{f'}$  and  $\sigma_0^{NC}$  are the cross sections of single  $\pi^0$  production from the neutrino magnetic moment and the standard model NC interac-

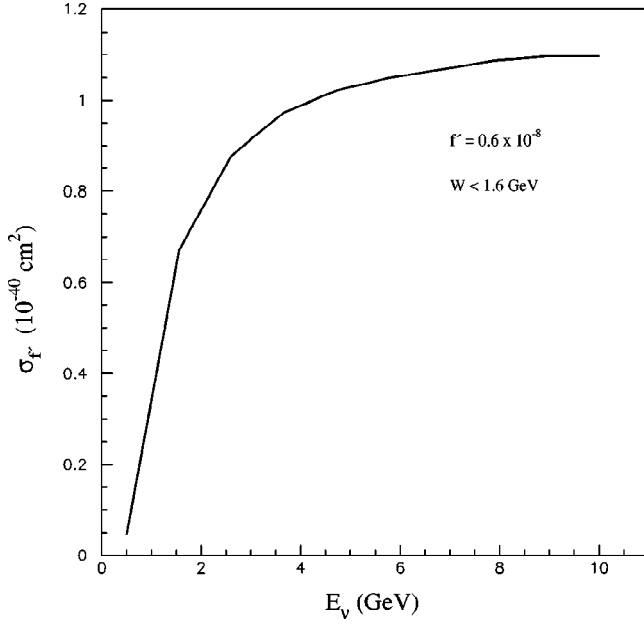


FIG. 2. The cross section generated by the neutrino transition magnetic moment as a function of the incident neutrino energy for the hadronic invariant mass less than  $1.6 \text{ GeV}/c^2$ .  $f'$  is taken to be  $0.6 \times 10^{-8}$ .

tions, respectively. The reason is that  $e$ -like events (CC) are not affected by the presence of the neutrino magnetic moment. In Fig. 3, we plot this ratio

$$r_{f'/NC} = \frac{\sigma_{f'}}{\sigma_0^{NC}} \quad (42)$$

as a function of the incident neutrino energy  $E_\nu$  for  $f' = f'_0 \equiv 0.6 \times 10^{-8}$ .  $f'_0$  is defined as the value giving a similar contribution as the NC interaction. If  $f'$  is  $\epsilon$  times  $f'_0$ , Fig. 3 should be multiplied by a factor  $\epsilon^2$ . Note that the contributions from the transition magnetic moment and from the standard model NC do not mix in the process  $\nu + N \rightarrow \nu' + N'$  due the unmixable  $\gamma$  matrix structure among these two. However, for  $\nu + N \rightarrow \nu' + \Delta$  there are terms which mix these two contributions. This is because the number of indices in the form factors  $\Gamma_\mu$ , defined in  $\bar{\Delta} \Gamma_\mu N$ , can match that of the  $\gamma$  matrices by eliminating one index in  $\bar{\Delta}_\mu \Gamma_\nu$  by  $q_\mu$ . The dashed line corresponds to the case of no cut whereas the solid line corresponds to the hadronic invariant mass cut at  $1.6 \text{ GeV}/c^2$ . The current experimental result of the ratio  $R_{\pi^0/e}$  implies that the possible excess from 1 amounts to 0.13 from which we can obtain the constraint on the neutrino magnetic moment. For example, at  $E_\nu = 5 \text{ GeV}$  a constraint  $r_{f'/NC} \leq 0.13$  leads to

$$f' \leq 2.2 \times 10^{-9}. \quad (43)$$

The transition magnetic moment of this magnitude implies a muon neutrino and sterile neutrino mass matrix of the form

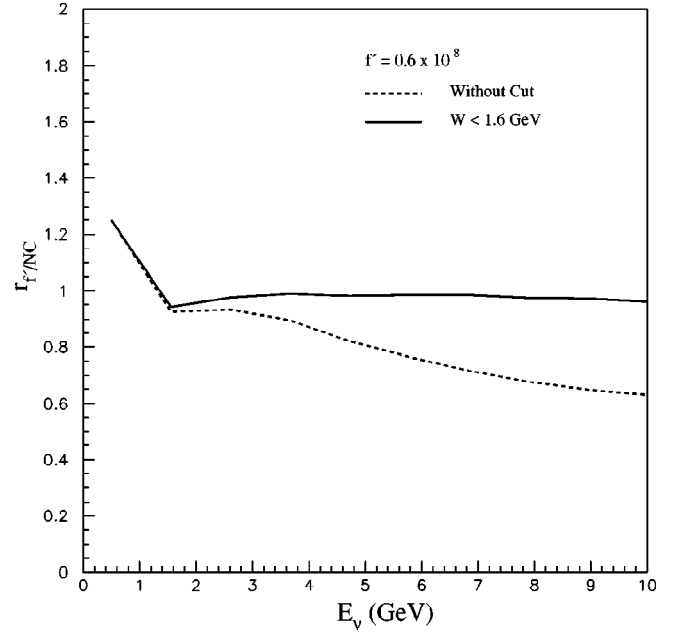


FIG. 3. Plots of  $r_{f'/NC}$  as a function of the incident neutrino energy for  $f' = 0.6 \times 10^{-8}$ . The dashed line corresponds to the case of no cut and the solid line corresponds to the invariant mass cut at  $1.6 \text{ GeV}/c^2$ .

$$M_{\nu_\mu \nu'} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}, \quad (44)$$

where  $m_{12} \sim m_{21}$  is roughly  $10^4$  times  $m_{11} \sim O(10^{-2}) \text{ eV}$ . Thus  $m_{12}$  is of order  $\leq 100 \text{ eV}$ . The diagonalization process should not change the mass of the muon neutrino drastically, i.e.,  $m_{22} \geq m_{12}^2/m_{11} \sim 1 \text{ MeV}$ . Therefore, a singlet neutrino at the intermediate scale with possible interactions beyond the standard model (scalar or gauge) can lead to a sizable transition magnetic moment.

The effects of the muon neutrino transition magnetic moment should be separated out toward a final determination of the muon neutrino oscillation to the tau neutrino or to a sterile neutrino. The most promising method is to study the energy distribution of the final  $\pi^0$ , since the kinematics for the magnetic moment is different from the NC interactions where the former has a  $1/q^2$  dependence in the differential cross section while the latter has no  $q^2$  dependence at low energy.

Indeed, if the transition magnetic moment is discovered by the measurement of the energy distribution, it will hint an intermediate scale physics. On the other hand, one can compare this anticipation to the earlier expectation that neutrinos must oscillate due to the belief that singlet fermions, the remnants of grand unification or the standard model superstring, would be present at the intermediate scale [26,27]. Similarly, if singlet neutrinos are present much above the eV scale, there may be a large transition magnetic moment which can be detected by future high statistics atmospheric neutrino experiments.

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## APPENDIX

We present the expressions of  $A_i^0$ ,  $i = N, NN\pi, P, S, D$  [20]:

$$A_\pi^0 = 3 \sqrt{\frac{1}{2}} G_F J^\lambda \sqrt{2} g_{NN\pi} \bar{u}(p') \gamma_5 u(p) \times \frac{2q_\pi^\lambda + q^\lambda}{(q_\pi + q)^2 - m_\pi^2} g_A' F_\pi, \quad (\text{A1})$$

$$A_N^0 = -3 \sqrt{\frac{1}{2}} G_F J^\lambda \sqrt{2} g_{NN\pi} \bar{u}(p') \times \left[ g_V' F_1 \gamma_\lambda + g_V' \frac{F_2}{2M_N} [\gamma_\lambda, \not{q}] - g_A' \frac{1}{3} F_A \gamma_\lambda \gamma_5 \right] \times \frac{\not{p}' + \not{q} + M_N}{(p' + q)^2 - M_N^2} \gamma_5 u(p), \quad (\text{A2})$$

$$A_{NN\pi}^0 = 3 \sqrt{\frac{1}{2}} G_F J^\lambda \sqrt{2} g_{NN\pi} \bar{u}(p') \times \frac{\not{p}_\Delta + M_N}{p_\Delta^2 - M_N^2} \left[ g_V' F_1 \gamma_\lambda + g_V' \frac{F_2}{2M_N} [\gamma_\lambda, \not{q}] - g_A' \frac{1}{3} F_A \gamma_\lambda \gamma_5 \right] u(p), \quad (\text{A3})$$

$$A_P^0 = -\frac{3}{2} \sqrt{\frac{1}{2}} G_F J^\lambda f_P \bar{u}(p') \frac{\not{p}_\Delta + M_P}{p_\Delta^2 - M_P^2} D_A^S g_A' \gamma_\lambda \gamma_5 u(p), \quad (\text{A4})$$

$$A_S^0 = \frac{9}{2} \sqrt{\frac{1}{2}} G_F J^\lambda f_S \bar{u}(p') \frac{\not{p}_\Delta + M_S}{p_\Delta^2 - M_S^2} \left[ g_V' G_1^S \gamma_\lambda - g_V' \frac{G_2^S}{2M_N} \right] \times [\gamma_\lambda, \not{q}] \gamma_5 + g_A' \frac{1}{3} G_A^S \gamma_\lambda u(p), \quad (\text{A5})$$

$$A_D^0 = -\frac{9}{2} \sqrt{\frac{1}{2}} G_F \frac{f_D}{m_\pi} \bar{u}(p') q_\pi^\mu \gamma_5 D_{\mu\rho} \times \left[ (q g^{\rho\lambda} - q^\rho \gamma^\lambda) g_V' \frac{H_3^S}{M_N} + (p_\Delta \cdot q g^{\rho\lambda} - q^\rho p_\Delta^\lambda) g_V' \frac{H_4^S}{M_N^2} - (p \cdot q g^{\rho\lambda} - q^\rho p^\lambda) g_V' \frac{H_5^S}{M_N^2} - g^{\rho\lambda} g_A' \frac{1}{3} H_A^S \gamma_5 \right] u(p), \quad (\text{A6})$$

where  $G_F = 1.023 \times 10^{-5} M_N^{-2}$  is the Fermi constant,  $g_{NN\pi}/4\pi = 14.8$  is the  $NN\pi$  coupling constant,  $M_N$  is the nucleon mass,  $m_\pi$  is the pion mass, and  $M_R$  the mass of the generic resonance  $R$ . We also assume  $M_A = 0.65 \text{ GeV}/c^2$  [ $M_A = 1.0 \text{ GeV}/c^2$  for the axial mass of the  $P_{33}(P_{11}, S_{11}, D_{13})$ ]. The generic vector form factor in the amplitudes Eqs. (A1)–(A3) is given by the general formula

$$V(t) = V^p(t) + V^n(t), \quad (\text{A7})$$

where  $V^p(V^n)$  is the electromagnetic form factor with a proton (neutron) as target.

With regard to the pion and nucleon form factors, we use the following usual forms:

$$F_\pi(t) = \frac{1}{1 - t/0.47 \text{ GeV}^2}, \quad (\text{A8})$$

$$F_1^V(t) = \left( 1 - \frac{3.7t}{4M_N^2 - t} \right) \frac{1}{(1 - t/0.71 \text{ GeV}^2)^2}, \quad (\text{A9})$$

$$F_2^V(t) = 1.855 \left( 1 - \frac{t}{4M_N^2} \right)^{-1} \frac{1}{(1 - t/0.71 \text{ GeV}^2)^2}, \quad (\text{A10})$$

$$F_A(t) = 1.23 \frac{1}{(1 - t/0.81 \text{ GeV}^2)^2}, \quad (\text{A11})$$

where  $t = Q^2$  is the moment transfer and the axial form factor is characterized by  $F_A(0) = 1.23$ , which is derived from neutron decay, whereas  $M_A = 0.90 \text{ GeV}/c^2$  is perfectly compatible with an overall fit to neutrino experiments. The form factors  $G_i$  and  $H_i$  are explicitly given in Ref. [20].

The axial form factors in the amplitudes Eqs. (A1)–(A3) are taken to be the general formula

$$A(t) = \frac{A(0)}{(1 - t/M_A^2)^2}, \quad (\text{A12})$$

where  $M_A = 0.65 \text{ GeV}/c^2$  for  $C_5^A$  and  $M_A = 1.0 \text{ GeV}/c^2$  for the other resonances.



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